**Homework 1- Heat Transfers**

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Deadline to give me back the homework:

Student name (Pinyin):

Student name (Chinese):

Student number:

**Ex. 1. The pressure of an ideal gas and the total kinetic energy of its molecules.**

**We want to describe the pressure of an ideal gas. The x-direction is the direction perpendicular to the wall. The molecules of the ideal gas have collisions with the wall. To simplify the problem, we consider first that all the molecules has the same -velocity. We assume the number of molecules per unit volume is the same everywhere, equals to ( is the total number of molecules in the volume ).**

**a) Describe the number of molecules in the cylinder shown on the figure of volume where is an arbitrary area of the wall, the distance travelled in the -direction by a molecule during , in terms of .**

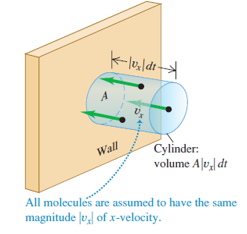
**(b) Half of the molecules move toward the wall and half of the molecules move away, which means that the number of collisions between the molecules and the wall occurring during is . The total change of momentum during correspond to:, where is the change of momentum of one molecule during one collision. Describe in terms of the mass of the molecule and . Thus, describe in terms of , .**

**c) Describe the force exerted by the molecules on the wall during in terms of . Why we don’t consider the change of momentum in y-direction and the z-direction in the expression of ?**

**d) The pressure is defined as the force exerted by the gas molecules on the wall per unit area. Describe the pressure in terms of . Also, we have considered that all the molecules have the same x-velocity which is not right. Describe the pressure in terms of where is the average value of .**

**e) Conclude by the description of the pressure in terms of the total kinetic energy of the molecules ( you don’t have to demonstrate that ).**

***Please to do the demonstration when it is needed. Avoid to just give a result obtained in lecture.***



**Solution:**

a) is the number of molecules in the cylinder shown on the figure of volume where is an arbitrary area of the wall, the distance travelled in the -direction by a molecule during .

b)

The number of collisions between the molecules and the wall occurring during is

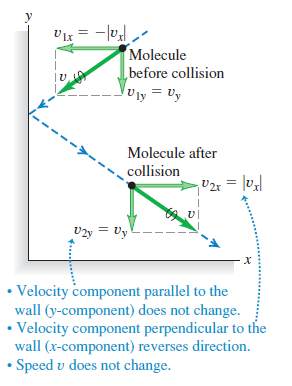
The total change of momentum during is:

The change of momentum during the collision of a molecule with the wall is:

We obtain

c) Force exerted by the molecules on the wall during is:

There is no change of momentum in the y-direction and the z-direction, the only change of momentum is in the direction perpendicular to the wall because the wall exerts on the molecule a force perpendicular to its surface (change of momentum because of gravity is not considered):



d) The pressure is defined as force exerted by the gas molecules on the wall per unit area:

We obtain this result because we have considered the -velocity is the same for all the molecules, which is wrong. A better description is then:

e)

**Comment:** although you don’t have to remember this full demonstration, you must to be able to do it, and you must to know what is a pressure (the force exerted by the molecules on the wall per unit area), the kinetic energy of a molecule and the total kinetic energy of all the molecules, the momentum, etc.

**Ex. 2. The most probable velocity in the Maxwell-Boltzmann distribution**

**We consider the Maxwell-Boltzmann distribution of an ideal gas:**

**where is a velocity of a molecule, the mass (the same for all the molecule of the ideal gas), is the temperature, is the Boltzmann constant.**

**a) Describe the distribution of the Kinetic energy .**

**b) As has a maximum value at the most probable velocity , the distribution of the kinetic energy has a maximum value, corresponding to the most probable kinetic energy , and corresponding to . First, please to describe then please to find for which value of kinetic energy corresponding the velocity the most probable (in terms of ).**

**c) Describe the most probable velocity in terms of (you have to demonstrate your result).**

**d) Compare with the result obtained in lecture about the root mean square velocity (you don’t have to demonstrate it).**

**Solution**

a)

The kinetic energy of a molecule of mass at velocity is defined by:

We obtain:

We obtain:

b)

The distribution of kinetic energy is (symbol “” means “proportional to”.):

Its derivative is such as:

I remind use about derivative of a product of two functions (“ u’ “ means “derivative of u”):

About the derivative of a function such as :

About the derivative of a function such as where is a constant:

We obtain

There is a maxima of for , i.e.

c)

The most probable velocity is the velocity such as:

d)

The root mean square velocity is:

We can observe that both characteristics velocities and are proportional to each other and are proportional to

**Ex. 3**

**A 20.0-L tank contains of helium at temperature . The molar mass of helium is**

**(a) How many moles of helium are in the tank?**

**(b) What is the pressure in the tank, in pascals and in atmospheres (the Helium has the behavior of an ideal gas)?**

**Use:**

**Solution**

a.The number of moles is

b.

Ideal gas law:

The temperature in Kelvin unit is:

The pressure in Pa unit is:

The pressure in atmosferes unit is:

**Ex. 4.**

**A 3.00-L tank contains air at 3.00 atm and (the air is assumed to have the behavior of an ideal gas). The tank is sealed and cooled until the pressure is 1.00 atm.**

**(a) What is the temperature then in degrees Celsius? Assume that the volume of the tank is constant.**

**(b) If the temperature is kept at the value found in part (a) and the gas is compressed, what is the volume when the pressure again becomes 3.00 atm?**

**Solution**

a.The air is cooled at constant volume from temperature and pressure to temperature and pressure . The number of moles and the volume is constant’

The temperature in is then :

b. The gas is compressed at constant temperature from pressure and volume to pressure and volume .

**Ex. 5.**

**Smoke particles in the air typically have masses of the order of . The Brownian motion (rapid, irregular movement) of these particles, resulting from collisions with air molecules, can be observed with a microscope. Thus, smoke particles have the same behavior than air molecules, and can be described approximately has a molecules of ideal gas, although they are at the solid state. Find the root-mean-square velocity of Brownian motion for a particle with a mass of in air at temperature equals 300 K.**

For the Boltzmann constant, use:

**Solution**

The root mean square velocity is:

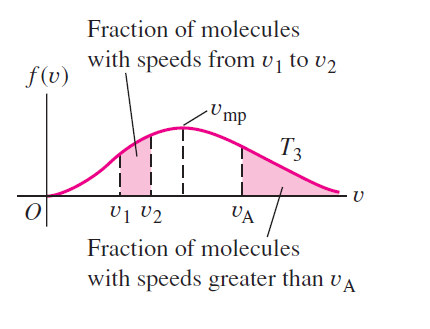
where is the Boltzmann constant, is the temperature and is the mass of a particle of smoke which has the same behavior than an ideal gas. The root mean square velocity is:

**Ex. 6**. **The average velocity of an ideal gas is described by:**

**where is the Maxwell-Boltzmann distribution of the velocities, is the total number of molecules, describes the number of molecules which the velocity is in the range . We are interested by the molecules which the velocity is in the range . is the number of molecules which the velocity is in the range .**

**a]Please to describe the average velocity of the molecules in this range in terms of , (take care you cannot put outside the integrals, and you don’t have to calculate these integrals).**

**b) There are molecules which the velocity is greater than In a similar way, please to describe the average velocity of the molecules which the velocity is greater than a velocity , in terms of**



**Solution**

a) The average velocity of the molecules in the range is described by:

Take care that the average velocity for the molecules in the range **cannot be** described by

To understand this, you must to think about what is an average, so the number of molecules which the velocity is in the range must appear.

which describe the number of molecules which the velocity is in the range is valid for any range considered.

The number of molecules which the velocity is in the range is:

We obtain the average velocity of the molecules which the velocity is in the range:

It is not possible to go further in the description. cannot been put outside the integrals. The values of both integrals depend to which range is considered.

b)

The number of molecules which the velocity is greater than is,

The average velocity of the molecules which the velocity is greater than is:

**Ex. 7.** **Number of degrees of freedom and internal energy of different kinds of gas** **(you should read the ppt of lesson 3 before to do this exercise).**

**a)Considering an ideal gas, which could be mono-atomic or polyatomic (all molecules of the gas are identical) , describe its internal energy in terms of its total number of degrees of freedom , its number of molecules and its temperature (you don’t have to demonstrate it). Then describe in terms of and the number of moles of gas molecules (demonstrate it from your previous reply).**

**b) Complete the table for different kinds of gas. is the number of degrees of freedom for the translational motion. is the number of degrees of freedom for the rotational motion, the total number of degree of freedom if we consider both translational and rotational motion, i.e. (no vibrational motion is considered here). is the average translational kinetic energy of a gas molecule (to describe in terms of temperature). is the average rotational kinetic energy of a gas molecule (to describe in terms of temperature). is the internal energy of the gas if we consider both translational and rotational motion (a mono-atomic gas has no rotational degrees of freedom). Calculate the internal energy of one mole of gas at temperature .**

**c) The number of degrees of freedom to consider depends strictly to the temperature of the gas and not only the gas considered. For instance, bellow 100 K, the molecules don’t have a rotational motion. Between 100 K and 600 K, the molecules have a translational and a rotational motion. Above 600 K, they have a translational motion, a rotational motion and also a vibrational motion where the atoms of have a simple harmonic motion (they can be seen as connected by a spring), with 2 degrees of freedom for the vibrational motion. Please to give the expression of the internal energy of one mole of in terms of temperature when its temperature is below 100 K, between 100 K and 600 K and above 600 K.**

**About the ideal law constant, use**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Gas |  |  |  |  |  | of 1 mol of gas at |
| (Helium) |  |  |  |  |  |  |
| (Argon) |  |  |  |  |  |  |
| (dihydrogen, also named hydrogen) |  |  |  |  |  |  |
| (dioxygen, also named oxygen) |  |  |  |  |  |  |
| (carbon dioxide) |  |  |  |  |  |  |
| (water) |  |  |  |  |  |  |
| (ammoniac) |  |  |  |  |  |  |

**Solution:**

a) The internal energy of an ideal gas at temperature , of molecules, with degrees of freedom is:

The total number of gas molecules is where is the Avogadro’s number (the number of molecule in one mole of gas). The Boltzmann constant is: where is the ideal gas constant.

We obtain

The internal energy of an ideal gas is also described by:

b)

The average translational kinetic energy of a gas molecule is:

The average rotational kinetic energy of a gas molecule is

About the internal energy, we consider both translational and rotational motion, thus

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Gas |  |  | I |  |  | of 1 mol of gas at |
| (Helium) | 3 | 0 | 3 |  | 0 | 3652 J |
| (Argon) | 3 | 0 | 3 |  | 0 | 3652 J |
| (dihydrogen, also named hydrogen) | 3 | 2 | 5 |  |  | 6087 J |
| (dioxygen, also named oxygen) | 3 | 2 | 5 |  |  | 6087 J |
| (carbon dioxide) | 3 | 2 | 5 |  |  | 6087 J |
| (water) | 3 | 3 | 6 |  |  | 7304 J |
| (Ammoniac) | 3 | 3 | 6 |  |  | 7304 J |

Take care that molecules have a linear arrangement, thus it has the same number of degrees of freedom than the diatomic gas.

c)

For one mole of gas, i.e. :

Bellow 100 K, the molecules of don’t have rotational motion which is freezing out, i.e. . The internal energy of one mole of is:

Between 100 K and 600 K, the molecules have a rotational and a translational motion, but don’t have a vibrational motion. The internal energy of one mole of is:

Above 600 K, the molecules have a translational motion, a rotational motion and a vibrational motion with 2 degrees of freedom for the vibrational motion. Its number of degrees of freedom is .The internal energy of one mole of is:

**Ex. 8. Energy of an atom in an ideal crystal (you should read the ppt of lesson 3 before to do this exercise).**

**The atoms in a mono-atomic crystal are assumed to have vibrations similar to a simple harmonic motion (thus the atoms can be seen connected by springs, but the forces considered are Van Der Walls forces, similar to the electrostatic force between two point charges, they are much more complicated to describe and are related with the electric cloud of the atoms in the crystal). Unlike the diatomic gas molecules and similarly to mono-atomic gas molecules, these atoms in the crystal don’t have a rotational kinetic energy. They had 3 degrees of freedom for the translational motion (the atoms can move in the three directions).**

**a) Assuming you can describe the translational kinetic energy of the atoms in the ideal crystal as the translational kinetic energy of the molecules in a ideal gas, describe the average translational kinetic energy of an atom in the crystal in terms of temperature .**

**b) The atom in a crystal can be seen has a harmonic oscillator. Its average potential energy and average kinetic energy are equal (this is not asked to demonstrate in this exercise, you can try to verify it for a spring-mass system in simple harmonic motion in the -direction, if you wish). Deduce an expression of the average potential energy of the atom in the ideal crystal in terms of the temperature of the crystal.**

**c) The average total energy of the atom in the crystal is the sum of its average potential energy and its average kinetic energy. Describe the average total energy of an atom in the ideal crystal in terms of temperature . According to our model, describe the total energy of a crystal which have N atoms, in terms of and .**

**Solution:**

a)Each degree of freedom as kinetic energy ( is the Boltzmann constant, T is the temperature). There are 3 degrees of freedoms for the translational motion, i.e.

b)

The average kinetic energy has the same value than the average potential energy. For the atom in the ideal crystal:

Its average potential energy is then:

c)

The average total energy of an atom in the ideal crystal is:

The total energy of all the atoms in the ideal crystal is: